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RATES UNIVERSITY

01. Basic Financial Maths & Bond Pricing

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Contents

This short module will focus on the following basic financial mathematics calculations

- Simple Interest
- Compound Interest
- Interest Rate bases
- Annuities

I will illustrate each of the concepts with examples

Finally we will look at how Bond prices are calculated.

Simple Interest

Simple Interest is the method used when the amount of interest per period is calculated on the Initial Principal only.

Interest is thus not calculated on the accumulated interest earned to date (interest is not compounded)

Define the following variables

P = Principal
I = Total Simple Interest
S = Accumulated Value (Future Value of P)
r = simple interest rate per period
t = number of time periods

The periods are usually measured in years, and this is the convention we adopt.

The basic formula for simple interest is that

$$I = P \times r \times t$$

Simple Interest

It immediately follows that …

$$S = P\,(1 + r\,t)$$

Inverting this we can see that the discounting equation becomes

$$P = \frac{S}{(1 + r\,t)}$$

Example 1

To what amount would EUR 1,000 accumulate at 4.00% p.a. simple interest for 9 months?

$$\mathbf{S} = P\,(1 + r\,t)$$

$$= 1{,}000 \times (1 + 0.04 \text{ x } 0.75)$$

$$= \mathbf{1{,}030}$$

Simple Interest

Coupons are calculated on a simple interest basis …

Coupon Amount = Coupon Rate x Year Fraction

Year Fraction is calculated via 30/360, Act/360, Act/365, … bases

Example (a) Qtly coupon paid with a 5.00% nominal rate 30/360
Coupon Amt = 5.00% x 90/360 = 1.25%

Example (b) Semi coupon paid with a 7.50% nominal rate Act/360
Coupon Amt = 7.50% x 182/360 = 3.792%

Compound Interest

Compound Interest is the interest method used when interest amounts due are reinvested (thereby earning interest), rather than being paid periodically.

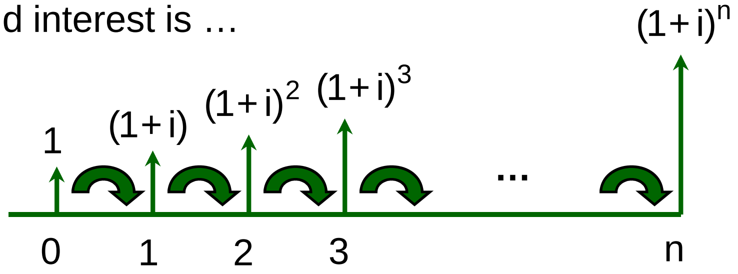
We define the following variables

P = Principal
S = Accumulated Value (Future Value of P)
i = interest rate per period
n = number of time periods

The time periods can be for instance, days, months, quarters, semi-annual periods …

Then the fundamental formula for compound interest is …

$$S = P\,(1+i)^n$$



so that

$$P = \frac{S}{(1+i)^n}$$

Interest is calculated at rate i per period.
It is then reinvested at the same rate,
accumulating as we go

Compound Interest

Example 2

To what amount would USD 2,000 accumulate at 6.25% annual compounding for 3 years?

$$\mathbf{S} = P(1+i)^n$$

$$= 2{,}000\,(1+0.0625)^3$$

$$= 2{,}398.93$$

Example 3

What is the Present Value of EUR 10,000 due in 5 years, where interest compounds at 5.50% annual?

$$\mathbf{P} = \frac{S}{(1+i)^n}$$

$$= \frac{10{,}000}{(1+0.055)^5}$$

$$= 7{,}651.34$$

Interest Rate Bases

In many compound interest situations, interest is compounded more frequently than annually.

For example, interest could be compounded …

- semi-annually
- quarterly
- monthly
- daily

Irrespective of the compounding frequency, interest rates are generally expressed as nominal annual rates. These need to be converted to an effective rate corresponding to the compounding period before they can be used in calculations.

For example …

- 8.00% nominal annual rate, compounded semi = effective rate of 4.00% every 6m
- 8.00% nominal annual rate, compounded quarterly = effective rate of 2.00% every 3m
- 8.00% nominal annual rate, compounded monthly = effective rate of 8%/12 = 0.67% each month

These rates are all different, and we need to be careful when applying them.

Interest Rate Bases

We define the following additional notation

m = frequency of compounding
j_m = nominal interest rate p.a. compounded m times per year
i = effective interest rate per period
j = effective annual interest rate

By definition

$$i = j_m / m$$

By considering the compounding of $1, m times per year, for 1 year, we have

$$(1+i)^m = (1+j_m/m)^m = (1+j)$$

Consequently

$$j = (1+j_m/m)^m - 1$$

Indeed we can convert between rates of different compound frequencies via …

$$(1+j) = (1+j_2/2)^2 = (1+j_4/4)^4 = (1+j_{12}/12)^{12} = (1+j_{365}/365)^{365}$$

Interest Rate Bases

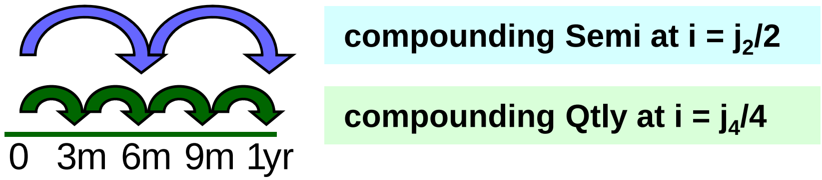
For example, if we want to convert from a 6.00% nominal annual rate to the equivalent nominal semi rate, we use

$$(1+j) = (1+j_2/2)^2$$

so that …

$$j_2 = 2\times[(1+j)^{0.5} - 1] = 2\times[(1.06)^{0.5} - 1] = 5.913\% \text{ Semi}$$

Likewise, converting $j_4 = 5.00\%$ nominal quarterly into the equivalent semi rate, we proceed via



compounding Semi at i = $j_2/2$

compounding Qtly at i = $j_4/4$

$$(1+j_2/2)^2 = (1+j_4/4)^4$$

or

$$(1+j_2/2) = (1+j_4/4)^2$$

and

$$j_2 = 2\times[(1+j_4/4)^2 - 1] = 2\times[(1+0.05/4)^2 - 1] = 5.031\% \text{ Semi}$$

**Note that 5.00% Qtly grosses up to 5.031% Semi.
5.00% Qtly is a higher rate than 5.00% Semi since compounding Qtly means interest is re-invested earlier.**

Continuous Compounding

It is sometimes convenient to allow for instantaneous, or continuous compounding.

This occurs when the frequency of re-investment approaches ∞.

Continuous compounding has nice mathematical properties, and is frequently encountered in various option formulae.

For this reason we briefly introduce it here.

It is easy to prove that the following mathematical relationship holds …

$$\lim_{n \to \infty} (1 + r/n)^n = e^r$$

From this, we find

$$e^c = (1 + j) = (1 + j_2/2)^2 = (1 + j_4/4)^4 = (1 + j_{12}/12)^{12} = (1 + j_{365}/365)^{365}$$

where c is the continuously compounded rate.

Taking logarithms (base e) both sides gives

$$c = \ln(1 + j) = 2 \times \ln(1 + j_2/2) = 4 \times \ln(1 + j_4/4) = 12 \times \ln(1 + j_{12}/12) = \ldots$$

Continuous Compounding

For example, converting a 6.00% nominal annual rate to the equivalent continuous rate, we find

$$c = \ln(1+j) = \ln(1+0.06) = 5.827\%$$

Similarly, converting a 5.25% Quarterly rate to the equivalent continuous rate,

$$c = 4 \times \ln(1+j_4/4) = 4 \times \ln(1+0.0525/4) = 5.216\%$$

Finally, converting a 5.00% daily rate to the equivalent continuous rate,

$$c = 365 \times \ln(1+j_{365}/365) = 365 \times \ln(1+0.0500/365) = 4.9996\%$$

Note that continuous rates are almost identical to daily rates

If we discount cashflows using continuous compounding we use the fact that

$$e^{-ct} = \frac{1}{(1+j)^t} = \frac{1}{(1+j_2/2)^{2t}} = \frac{1}{(1+j_4/4)^{4t}} = \ldots$$

where t is measured in years

So for example, discounting a cashflow of EUR 100,000 occuring in 3 years at a continuously compounded rate of 5.50% leads to a Present Value of …

$$P = e^{-ct} \times 100{,}000 = e^{-0.055\times 3} \times 100{,}000 = 84{,}789.97$$

Annuities

An annuity is a portfolio of identical cashflows that occur at regular points in time.

The most obvious example of an annuity are the coupons on a fixed rate Bond.

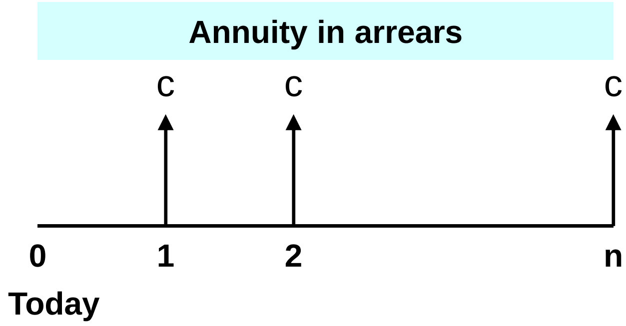
If we assume a flat (constant) discount rate we can derive a simple expression for the value of an annuity

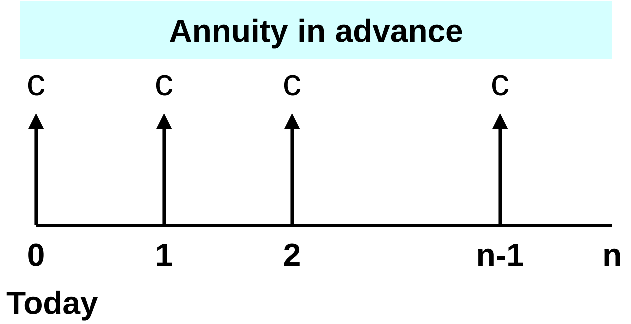
There are 2 basic types

- Annuity in arrears (the most common type)
- Annuity in advance

In an arrears annuity it is assumed that the common cashflow is paid at the end of each payment period.

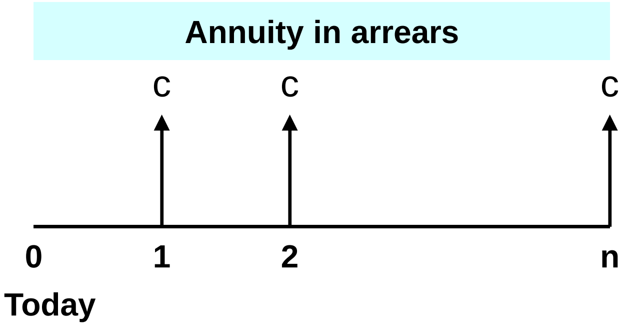
In an advance annuity it is assumed that the common cashflow is paid at the start of each payment period.





Annuities

We begin by looking at the annuity in arrears.


Annuity in arrears

We assume a flat discount rate i which applies for each of the n periods.

Note that i is not necessarily an annualised rate.
i is the discount rate per period, and periods can be monthly, quarterly, semi, …

Letting P denote the price, and applying compound interest to discount the cashflows c at a rate of i per period gives us

$$P = \frac{c}{(1+i)^1} + \frac{c}{(1+i)^2} + \frac{c}{(1+i)^3} + \ldots + \frac{c}{(1+i)^n}$$

If we define $v = \frac{1}{(1+i)}$

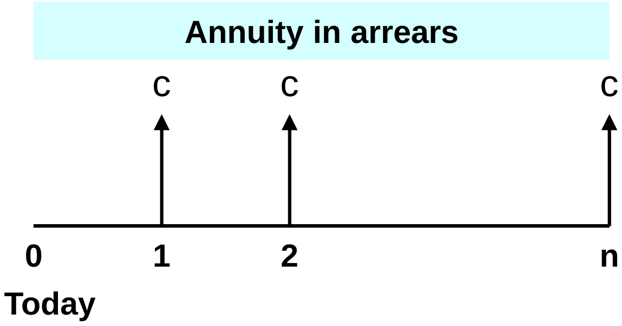
this becomes

$$P = c \bullet v^1 + c \bullet v^2 + c \bullet v^3 + \ldots + c \bullet v^n$$

or

$$P = \frac{c \bullet (1 - v^n)}{i}$$

Annuities



For those who prefer to see that result derived, we start with

$$P = c \bullet v^1 + c \bullet v^2 + c \bullet v^3 + \ldots + c \bullet v^n \qquad (1)$$

From this we see

$$\frac{P}{v} = c + c \bullet v^1 + c \bullet v^2 + \ldots + c \bullet v^{n-1} \qquad (2)$$

Subtracting (1) from (2) gives …

$$P \bullet \left(\frac{1}{v} - 1\right) = (c - c \bullet v^n) = c \bullet (1 - v^n) \qquad (3)$$

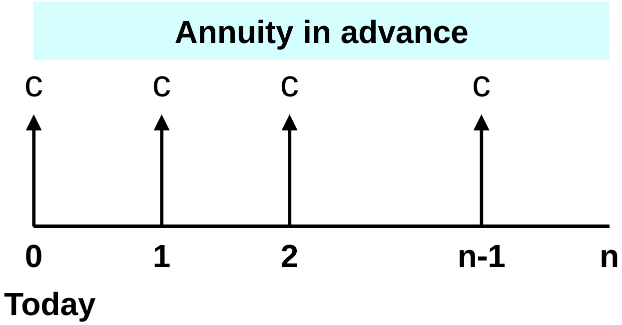
But

$$\frac{1}{v} - 1 = (1 + i) - 1 = i$$

and so (3) gives

$$P = \frac{c \bullet (1 - v^n)}{i}$$

Annuities



We now look at annuities in advance.

Again i is the discount rate per period.

This time, letting P* denote the Price …

$$P^* = c + \frac{c}{(1+i)^1} + \frac{c}{(1+i)^2} + \dots + \frac{c}{(1+i)^{n-1}} \qquad (4)$$

Note that each cashflow is now discounted by 1 less period as they now occur in advance.

Indeed, P* and P (the annuity in arrears Price) are related by …

$$\frac{P^*}{(1+i)} = \frac{c}{(1+i)^1} + \frac{c}{(1+i)^2} + \dots + \frac{c}{(1+i)^n} = P$$

so

$$P^* = P \times (1+i) = \frac{c \bullet (1 - v^n)}{i/(1+i)}$$

or

$$P^* = \frac{c \bullet (1 - v^n)}{(1 - v)}$$

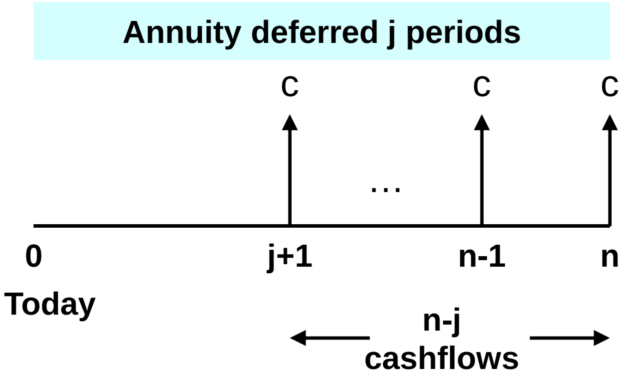
Deferred Annuities

Finally, we look at a deferred annuity.

Assuming the annuity this time consists of (n-j) cashflows c, with

- the first cashflow at time j+1
- the last cashflow at time n

This is a standard annuity in arrears, deferred j periods


Annuity deferred j periods

Clearly this annuity is a standard n period annuity in arrears less a standard j period annuity in arrears.

Hence

$$P_{def} = \frac{c}{(1+i)^{j+1}} + \frac{c}{(1+i)^{j+2}} + \dots + \frac{c}{(1+i)^{n}}$$

and

$$P_{def} = \frac{c \bullet (1 - v^{n})}{i} - \frac{c \bullet (1 - v^{j})}{i} = \frac{c \bullet (v^{j} - v^{n})}{i}$$

or

$$P_{def} = \frac{c \bullet v^{j} \bullet (1 - v^{n-j})}{i}$$

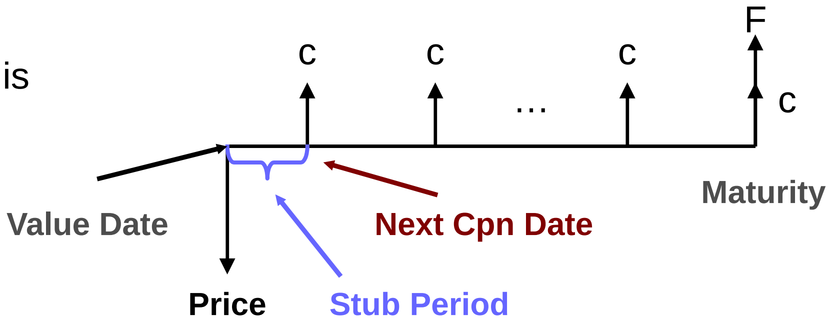
Bond Pricing

Having calculated a variety of formulas for various annuities, we can now look at the pricing of a standard fixed rate Bond.

A fixed rate Bond with Face Value F and Coupon Rate R is nothing more than the sum of

- an annuity of fixed coupons c
- a single zero coupon flow F at Maturity

Fixed Rate Bond



Note a full coupon is paid at the Next Cpn Date even though there is a short stub period to that Date.

where $c = F \times R \times DayCount$

and DayCount = 1 for Annual coupons
= ½ for Semi coupons
= ¼ for Qtly coupons

The pricing methodology assumes that we discount all Bond cashflows at a flat yield y, where y is expressed as a nominal annual rate.

This means that the annuity formulae we have already seen can be used to price the coupon flows.

If we price a semi-annual Bond, we would need to apply the annuity formulas, but using

$i = \frac{1}{2} y$ as the discount rate per period

Similarly $i = y$ if the Bond has annual coupons

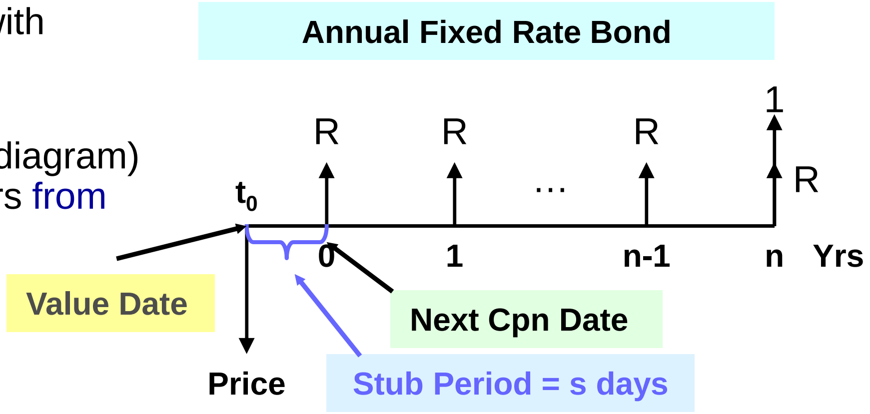
Bond Pricing – Annual Coupons

Consider then the pricing of an annual fixed rate Bond with coupon rate R and annual yield y.

Assume that the next coupon date (denoted by 0 in the diagram) is s days from the Value Date , and that there are n years from the next coupon date until Maturity.

Using a unit Bond Notional (F = 1) we have

$$c = R$$



We start by pricing, value the next coupon date, the remaining (n+1) coupons

Value next coupon date, the (n+1) annual coupons have value …

$$C_{next} = R + \frac{R}{(1+y)^1} + \frac{R}{(1+y)^2} + \frac{R}{(1+y)^3} + \ldots + \frac{R}{(1+y)^n}$$

Using the formula for a basic annuity in arrears, this has value

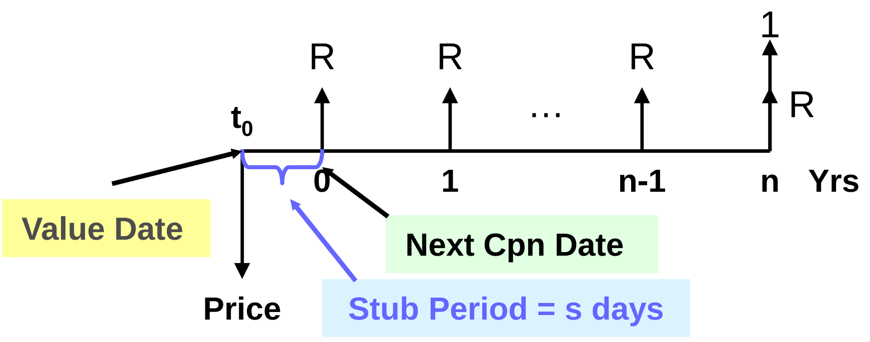
$$C_{next} = R + \frac{R \bullet (1 - v^n)}{y}$$

where

$$v = \frac{1}{(1+y)}$$

Bond Pricing – Annual Coupons

Annual Fixed Rate Bond



We then need to add, again value the next coupon date, the unit Notional at Maturity.

The unit Notional is worth, value the next coupon date

$$N_{next} = \frac{1}{(1+y)^n} = v^n$$

We now have the Bond Price, value the next coupon date

$$P_{next} = C_{next} + N_{next} = R + \frac{R \bullet (1 - v^n)}{y} + v^n$$

The final step is to discount the Price from the next coupon date back s days to our Value Date t_0

The usual approach is to use a discount factor to the next coupon date of $\frac{1}{(1+y)^{s/d}} = v^{s/d}$

where d = the number of days in the current coupon period (last coupon date to next coupon date)

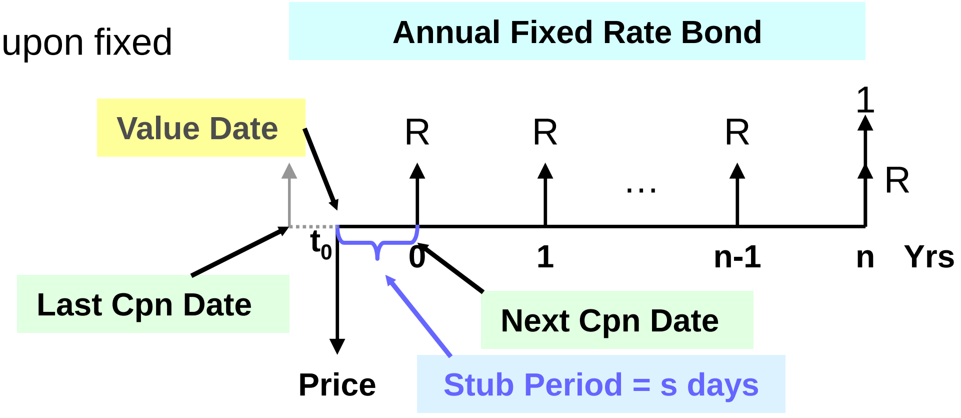
So, our Bond Price value t_0 is $P = v^{s/d} \bullet P_{next}$

or $$P = v^{s/d} \bullet [R + \frac{R \bullet (1 - v^n)}{y} + v^n]$$

Bond Pricing – Accrued Interest

The Price we have just calculated for an annual coupon fixed rate Bond, viz.

$$P = v^{s/d} \bullet [\, R + \frac{R \bullet (1 - v^n)}{y} + v^n \,]$$



where

R = coupon rate (annual)
y = yield (annual)
d = # days in current (annual) coupon period
s = # days from the Value Date to the next coupon date

and

$$v = \frac{1}{(1+y)}$$

is a so-called Dirty Price.

It is called that because it is “contaminated” by accrued interest.

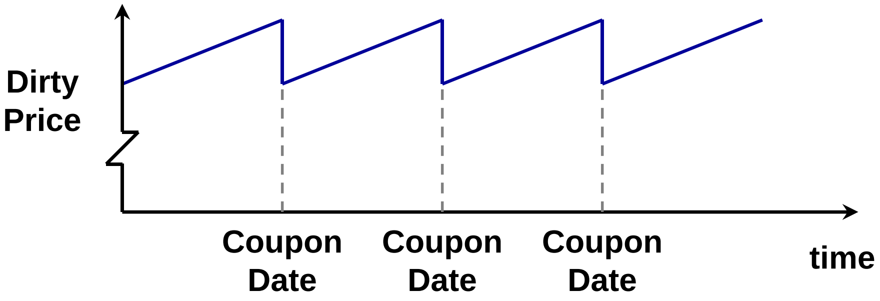
An investor who buys the Bond for value t_0 is entitled to a full coupon R on the next coupon date.

This is true irrespective of the length of the stub period. The investor is effectively receiving the coupon interest accrued from the Last Coupon Date until the Value Date, despite not having held the Bond during that period.

Bond Pricing – Accrued Interest

As a consequence of this accrued interest, the Dirty Price will fluctuate over time, even if yields do not change.

Indeed the Dirty Price typically has a classic “saw-tooth” graph.



The Dirty Price rises between coupon dates as interest accrues during the current period.

The Dirty Price then falls on each coupon date as coupons are paid and hence removed from the Dirty Price calculation.

$$P = v^{s/d} \bullet \left[R + \frac{R \bullet (1 - v^n)}{y} + v^n \right] \longrightarrow P = \left[\frac{R \bullet (1 - v^n)}{y} + v^n \right]$$

Dirty Price before the coupon paid | **Dirty Price just after the coupon paid**

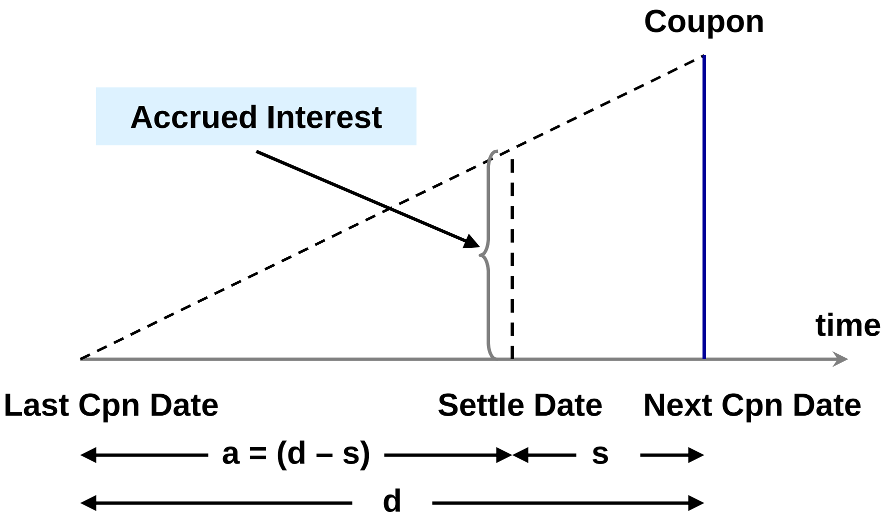
To enable investors to better monitor Bond Price movements due to yield changes, Bonds are typically quoted as a Clean Price, where

Clean Price = Dirty Price less Accrued Interest

Bond Pricing – Accrued Interest

Interest is assumed to accrue on a straight line basis … even though this is strictly speaking only approximately correct.

The Accrued Interest is calculated via …



Note that a,s and d are typically calculated on a 30/360 basis

The Accrued Interest is then

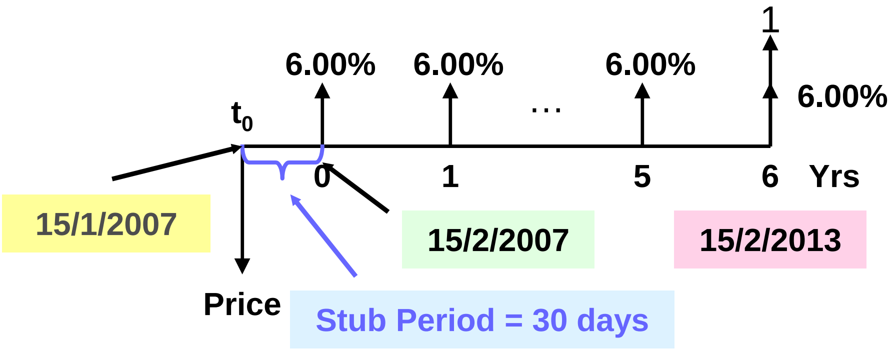
Accrued Interest = Coupon **x** (a / d)

where a = # days from Last Coupon Date to Settlement Date (30/360 basis)
d = # days in the current coupon period (30/360 basis)

Bond Pricing – Annual Coupon Example

We look at an example of pricing a fixed rate annual coupon Bond.

Annual Bond maturing 15 February 2013



Assume the following Bond data …

Settle Date	15 January 2007	
Maturity Date	15 February 2013	
Coupon	6.00%	annual
Yield	5.50%	annual

We use

$$\text{Dirty Price} = v^{s/d} \bullet \left[R + \frac{R \bullet (1 - v^n)}{y} + v^n \right]$$

where
$s = 30$ (1 month)
$d = 360$ (12 mths)
$a = 330$ (11 mths)

$$v = \frac{1}{(1+y)} = \frac{1}{(1+0.055)} = 0.9479$$

$$\text{Dirty Price} = 0.9479^{30/360} \bullet \left[6.00\% + \frac{6.00\% \bullet (1 - v^6)}{5.50\%} + 0.9479^6 \right] = 0.9479^{30/360} \bullet 108.498\% = 108.015\%$$

$$\text{Accrued Interest} = 6.00\% \bullet \frac{330}{360} = 5.500\%$$

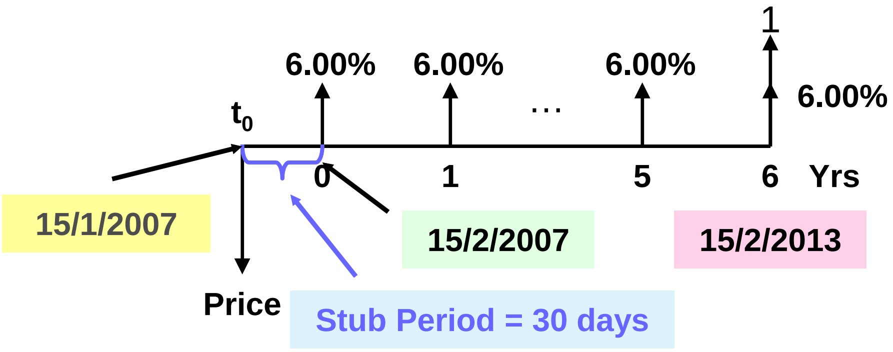
Clean Price = 108.015% - 5.500% = 102.515%

Prices above 100 since coupon > yield

Bond Pricing – Annual Coupon Example

If we re-price the same Bond with a yield this time of 6.25%

Annual Bond maturing 15 February 2013



Settle Date	15 January 2007	
Maturity Date	15 February 2013	
Coupon	6.00%	annual
Yield	6.25%	annual

We now have

$$v = \frac{1}{(1+y)} = \frac{1}{(1+0.0625)} = 0.9412$$

$$\text{Dirty Price} = 0.9412^{30/360} \bullet [\,6.00\% + \frac{6.00\% \bullet (1 - v^6)}{6.25\%} + 0.9412^6\,] = 0.9412^{30/360} \bullet 104.780\% = 104.252\%$$

$$\text{Accrued Interest} = 6.00\% \bullet \frac{330}{360} = 5.500\%$$

Clean Price = 104.252% - 5.500% = 98.752%

Prices below 100 since coupon < yield

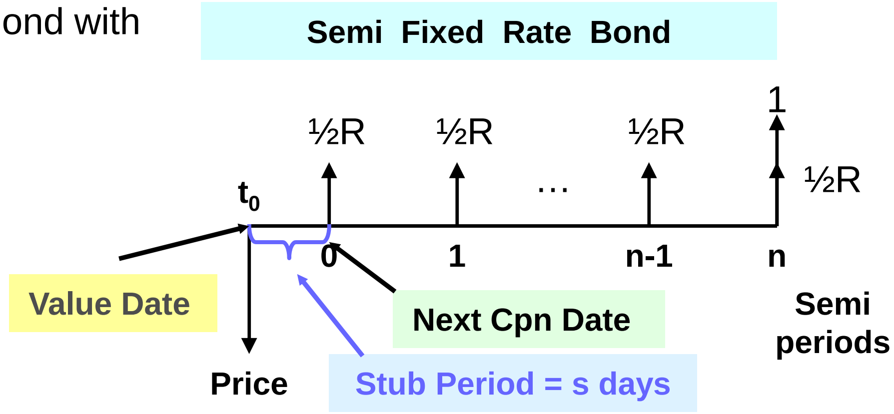
Bond Pricing – Semi Coupons

We now look at the pricing of a semi-annual fixed rate Bond with coupon rate R and semi-annual yield y.

Now coupons occur every 6 months.

Each coupon is now 50% of the quoted annualised coupon rate, so

$$c = \tfrac{1}{2} R$$



We assume now that there are n semi-annual periods from the Next Coupon Date till Maturity

Value next coupon date, the (n+1) remaining semi-annual coupons have value …

$$C_{\text{next}} = \tfrac{1}{2}R + \frac{\tfrac{1}{2}R}{(1+\tfrac{1}{2}y)^1} + \frac{\tfrac{1}{2}R}{(1+\tfrac{1}{2}y)^2} + \frac{\tfrac{1}{2}R}{(1+\tfrac{1}{2}y)^3} + \dots + \frac{\tfrac{1}{2}R}{(1+\tfrac{1}{2}y)^n}$$

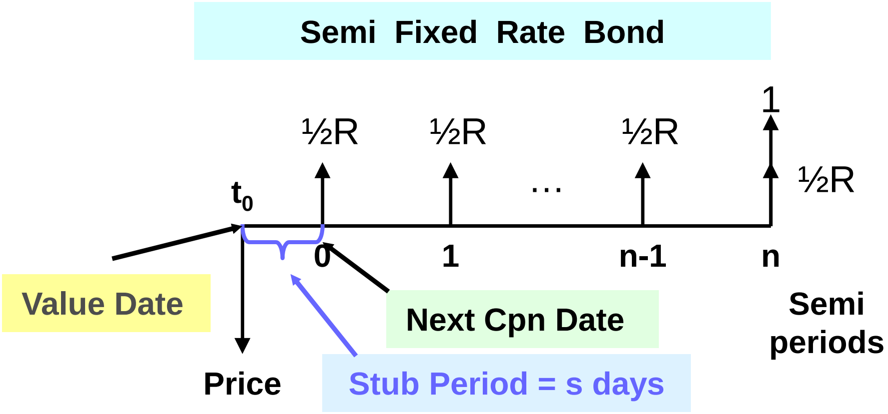
Using the formula for a basic annuity in arrears, this has value

$$C_{\text{next}} = \tfrac{1}{2}R + \frac{\tfrac{1}{2}R \bullet (1 - v^n)}{\tfrac{1}{2}y}$$

where

$$v = \frac{1}{(1+\tfrac{1}{2}y)}$$

Bond Pricing – Semi Coupons



Semi Fixed Rate Bond

Value Date

Next Cpn Date

Stub Period = s days

We then need to add, again value the next coupon date, the unit Notional at Maturity.

The unit Notional is worth, value the next coupon date

$$N_{next} = \frac{1}{(1+\frac{1}{2}y)^n} = v^n$$

We now have the Bond Price, value the next coupon date

$$P_{next} = C_{next} + N_{next} = \tfrac{1}{2}R + \frac{\frac{1}{2}R \bullet (1 - v^n)}{\frac{1}{2}y} + v^n$$

The final step is again to discount the Price from the next coupon date back s days to our Value Date t_0

This time we use a discount factor to the next coupon date of $\frac{1}{(1+\frac{1}{2}y)^{s/d}} = v^{s/d}$

where d = the number of days in the current semi coupon period (30/360 basis)

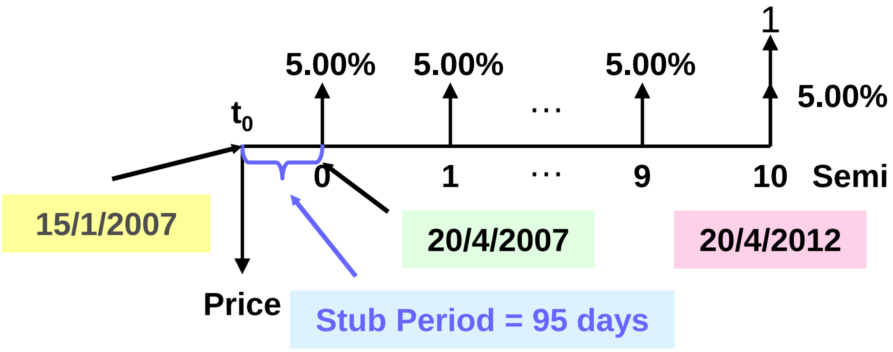
So, our Bond Price value t_0 is $P = v^{s/d} \bullet P_{next}$

or $$P = v^{s/d} \bullet [\, \tfrac{1}{2}R + \frac{\frac{1}{2}R \bullet (1 - v^n)}{\frac{1}{2}y} + v^n \,]$$

Bond Pricing – Semi Coupon Example

We look at an example of pricing a fixed rate semi-annual coupon Bond.

Semi Bond maturing 20 April 2012



Assume the following Bond data …

Settle Date	15 January 2007	
Maturity Date	20 April 2012	
Coupon	5.00%	semi-annual
Yield	5.25%	semi-annual

We use

$$\text{Dirty Price} = v^{s/d}\bullet\left[\tfrac{1}{2}R + \frac{\tfrac{1}{2}R\bullet(1-v^{n})}{\tfrac{1}{2}y} + v^{n}\right]$$

where
n = 10 (semi periods)
s = 95 (= 3x30 + 5)
d = 180
a = 85

$$v = \frac{1}{(1+\tfrac{1}{2}y)} = \frac{1}{(1+0.02625)} = 0.9744$$

$$\text{Dirty Price} = 0.9744^{95/180}\bullet\left[\,2.50\% + \frac{2.50\%\bullet(1-v^{10})}{2.625\%} + 0.9744^{10}\,\right] = 0.9744^{95/180}\bullet 101.413\% = 100.036\%$$

$$\text{Accrued Interest} = 2.50\%\bullet\frac{85}{180} = 1.181\%$$

Note that this is a % of the actual coupon of 2.50%

Clean Price = 100.036% - 1.181% = 98.855%

Prices below 100 since coupon < yield

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